Linear Programming Applied to Cost Minimizing Farm Management Strategies

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This paper illustrates use of minimum cost linear programming techniques with reference to a Great Plains wheat farm. It compares the optimum organization of enterprises developed by minimum cost and profit maximizing strategies of management on the same farm and discusses interrelationships between results of these two models. In general, the minimum cost technique may be used to provide the research worker with knowledge of a wider range of possible outcomes than profit maximizing models used alone and hence may be more useful in a predictive sense than profit maximizing techniques. Application of the minimum cost technique to other problems to which it is especially well suited is also discussed.

Profit maximization is not always the primary goal of farm managers. Many are cost conscious. Others demand sure and sometimes substantial returns before they will risk additional operating capital for certain farming practices or enterprises. In the Great Plains, farmers' problems are compounded by large investments, great variability in production, and a high level of risk and uncertainty which make it more difficult to follow rigorous decision-making criteria. As a result, we believe that many of these farmers do not follow a management strategy that would maximize their profits but instead follow a strategy of minimizing costs. For that reason, there is need for cost minimizing or other types of mathematical models to more accurately portray the real world decision-making process in this area.

This article will discuss a minimum cost model which seems to show considerable promise for the research worker interested in farms in the Great Plains and elsewhere. The objective will be to demonstrate the use of minimum cost decision-making criteria via linear programming techniques on a representative 1,600-acre farm in northwest Kansas. The resulting farm organization and returns will then be compared with an organization developed under the profit maximizing strategy of management on the same farm. Finally, several broad areas of applicability of the minimum cost analysis techniques will be discussed.

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Programming Models Used

The basic programming problem considered here is organizing production on a typical farm with a fixed investment in land. Organizations thus developed will be something less than long run in nature, since land investment is fixed. However, the general relationships presented are relevant over a wide range of resource fixity assumptions (that is, lengths of run).

Two different linear programming models were set up for this analysis. The profit maximizing model (used for comparisons) was of the usual type with the objective of determining the organization of enterprises maximizing the return over variable cost on the farm in question. The minimum cost model was developed from the profit maximizing model by substituting activity costs for activity returns in the objective function and by adding one additional constraint to assure that a desired level of return is achieved. The objective of the minimum cost model was to obtain various specified levels of return over variable cost using the least cost organization of production on the same farm. In all other respects, the two models used were identical to facilitate comparing the two different management strategies. The same data (coefficients, costs, and returns) were used in both models.

Possible revenue producing activities included were: (1) wheat on summer fallow, (2) grain sorghum (in three rotations) that could be either sold or fed to livestock, (3) barley that could be either sold or fed to livestock, (4) yearling steers wintered on crop stubble and wheat pasture, (5) yearling steers wintered on crop stubble and wheat pasture and grazed on available native pasture during the following summer, (6) a cow-calf enterprise, (7) a two litter sow system, and (8) renting out native pasture. Constraints were included in both models to: (1) limit rotations and crop acres to comply with accepted conservation and cropping practices in the area, (2) limit labor requirements by periods to the amount of operator labor available (use of additional hired labor was allowed when necessary), and (3) limit sources of income from spec-

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1 For a discussion of decision making in economics and a list of references on the subject, see Herbert A. Simon, "Theories of Decision Making in Economics and Behavioral Science," *Am. Econ. Rev.*, Vol. 49, June 1959.

2 A good example of a farm organization problem of this type may be found in the book by E. O. Heady and W. Candler, *Linear Programming Methods*, Ames, Iowa, Iowa State University Press, 1958, p. 119.

ified activities to maintain a wheat-grain sorghum type of farm. Specifically, limits were placed on the gross income for each activity to assure that wheat income is greater than sorghum income, wheat plus sorghum income is greater than 70 percent of total, and sorghum income is greater than 30 percent of total. These three constraints were placed in the model for use in another USDA study. The authors feel that these constraints do not affect the relevance of this paper.

Additional details about the models are omitted since they have no significant effect on the relationships that are discussed. However, for purposes of clarity, the following terms are defined:

a. Fixed costs include depreciation, interest, taxes, and insurance on land, buildings, improvements, and machinery, and other miscellaneous farm expenses not affected by size of enterprise or type of operation.

b. Variable costs include those costs which vary directly (have a linear relationship) with the size of the individual enterprise. They include such items as fuel, repairs, seed, and feed. Operator labor is not included as a cost but shares the returns.

c. Total cost is the sum of fixed costs and variable costs.

d. Return as used here refers to the "return over variable costs." This is the quantity maximized by the profit maximizing model.

e. Gross income includes crop sales, pasture rental, and livestock sales less purchase price.

Procedure Used with the Minimum Cost Model

The solution obtained using the minimum cost linear programming model is dependent upon the particular level of return specified by the researcher. A change in the level of specified return is always accompanied by (1) a change in the size of one or more of the activities in the solution, and possibly by (2) replacement of one or more of the activities. It follows that there is an infinite number of different solutions with the minimum cost model, each depending upon the particular level of return that is specified by the programmer. The problem then is to determine all changes in organization that occur in the optimum solution as the specified return is changed from a relatively low level to maximum returns.

As the level of specified return is changed, some of the activities in the optimum solution may be increasing, some may be decreasing, and some may be constant. Each time one of the decreasing activities reaches

\footnote{In this use, "activities" include disposal activities for unused resources as well as revenue producing activities.}
the zero level, it is replaced by a new activity in order to maintain optimality. The level of return that exists when an activity reaches zero and a new activity enters the solution is of particular interest. It is only at such points that the magnitude of the activities may be discontinuous (kinked). A perfect linear relationship exists in all parts of the solution between these points.

The levels of returns at which activities reach zero were determined by computing two solutions at different return levels but including the same activities. Then the linearity of the relationships enables the adjacent return levels at which activities reach zero levels to be derived. By the same reasoning, other solutions within this range may be easily derived from the two computed solutions. This process can be repeated until a number of such consecutive ranges are analyzed. This gives a complete picture of the behavior of the firm under the various return or output levels.

From a practical standpoint, farmers would be expected to give more consideration to the organizations found at points where the level of the activities are discontinuous and new activities are about to enter the solution than to organizations at other points. In general, as income is increased, an additional resource becomes scarce at each of the kinks. Also, it is at such points that each activity in the solution reaches a maximum. Beyond each of these points, obtaining slightly larger amounts of income involves increasing at least one activity from the zero level. At low levels some activities may not be practical.

Solutions to the Minimum Cost Model

Analysis of the 1,600-acre farm in northwestern Kansas using the minimum cost model and the procedure outlined above yielded the results presented in Figure 1. Shown in the figure are all possible minimum cost solutions for returns of $13,000 and above, gross income by source, as well as fixed costs, and total costs (fixed plus variable) associated with each level of return or operation.

An infinite number of solutions is presented in Figure 1, and the farm is organized differently for each level of return selected. This is because the relationships among the competing enterprises change as the level of returns from a given farm situation change. For low levels of returns, few, if any, resources are restrictive. The choice of enterprises included

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*For additional information, see Walter W. Garvin, *Introduction to Linear Programming*, New York, McGraw-Hill Co. Inc., 1960, pp. 220-231. Garvin discusses for the general case (1) how activities in the optimum solution may be expected to behave as one or more resources are varied, and (2) a more formal computing procedure for determining this behavior than has been used by the authors.
in the solution is made on the basis of the relative cost per dollar of return. At low levels of return, wheat on summer fallow is the primary enterprise on this farm. Enough grain sorghum is included to produce minimum sorghum income to meet the type of farm constraint.

As the level of return is increased, more and more resources are fully utilized, and the amount of return an enterprise furnishes per unit of these resources becomes increasingly important in determining the choice of enterprises. For example, wheat begins to decline in importance and is replaced to some extent by grain sorghum, which yields a higher return to the restricted land resource. At still higher return levels, the higher cost livestock enterprises are included since they obtain some return from as yet unused labor and pasture resources. Finally, as the maximum number of resources are utilized with the minimum cost model, the return of an enterprise per unit of each scarce resource becomes the primary criterion for selection of enterprises. On this farm, all livestock allowed by the type of farm constraint on income is included.

Hence, the organization of the farm under the minimum cost model changes as the specified level of return to be obtained is increased. Livestock enterprises, which have a relatively high cost per dollar of net return, are not included in solutions producing low returns. They are, how-
ever, included when the specified return is increased, since they furnish returns to resources that would otherwise be unused.\(^6\)

It should be reemphasized that the definition of gross income involves livestock sales minus purchase price and therefore the purchase price of livestock is not included as a variable cost in the activities. If the “gross sales” of livestock were used as the gross income and the purchase price were included as a variable cost, different solutions may be obtained. First, such livestock activities would have a higher cost per dollar of net return making them compare less favorably with cash gains as “low cost” activities. Second, livestock activities would be further restricted under this alternative definition of gross income unless the type of farm restraints on income were also redefined.

The farm organizations at the points designated by vertical lines on Figure 1 are of particular interest. These are solutions at three different levels of return at which a change in the organization occurs. It is at such points (there are, of course, more than three) that a farm probably would be operated in the real world since going to a higher level of return, under a minimum cost organization, involves adoption of a new activity instead of merely adjusting the size of activities already included.

Additional data on these three organizations are presented in Table 1. Organization 1 is a cash-grain organization, with wheat and grain sorghum furnishing nearly all income. The net return over variable costs from the farm under this system is $16,145, which produces a $4,500 return to operator labor and management after fixed costs are deducted.

Organization 2 shows the minimum cost organization at a higher level of return ($17,250) and includes a hog enterprise in addition to wheat and sorghum.

Organization 3 shows the organization, income, and expenses on the same farm with returns at $17,662. This is the highest return obtainable on this farm with the minimum cost model and the organization includes wheat, sorghum, hogs, and cattle.

Fixed costs are the same ($11,645) for each of the three organizations presented in the table since each represents the same fixed resource situation.\(^7\) In moving from organization 1 to organization 3, gross income increases by $3,920 and variable costs increase by $2,403. These changes increase the return to operator labor and management by $1,517.

\(^6\) It should be pointed out that the relationships between hogs and beef wintering in this solution are determined on the basis of rigorous price-production assumptions. From a practical standpoint, the differences would not be considered significant and the choice of which livestock enterprise to follow could be made on the basis of noneconomic factors.

\(^7\) These costs are fixed in the sense defined earlier.
Table I. Minimum cost organization and returns on a 1,600-acre northwest Kansas farm at three levels of operation

<table>
<thead>
<tr>
<th>Item</th>
<th>Wheat and sorghum (Organization 1)</th>
<th>Wheat, sorghum, and hogs (Organization 2)</th>
<th>Wheat, sorghum, hogs, and cattle¹ (Organization 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item</td>
<td>Acres² or head</td>
<td>Dollars³</td>
<td>Acres or head</td>
</tr>
<tr>
<td>Wheat</td>
<td>660</td>
<td>8,903</td>
<td>590</td>
</tr>
<tr>
<td>Grain sorghum (for sale)</td>
<td>488</td>
<td>8,902</td>
<td>440</td>
</tr>
<tr>
<td>Rent out native pasture</td>
<td>389</td>
<td>778</td>
<td>389</td>
</tr>
<tr>
<td>Hogs (sows farrowing 2 litters)</td>
<td>---</td>
<td>---</td>
<td>10</td>
</tr>
<tr>
<td>Winter steers</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Winter and summer steers</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Total gross income</td>
<td>18,583</td>
<td>21,235</td>
<td>22,503</td>
</tr>
<tr>
<td>Variable costs</td>
<td>2,438</td>
<td>3,983</td>
<td>4,841</td>
</tr>
<tr>
<td>Net return over variable costs</td>
<td>16,145</td>
<td>17,250</td>
<td>17,662</td>
</tr>
<tr>
<td>Fixed costs</td>
<td>11,645</td>
<td>11,645</td>
<td>11,645</td>
</tr>
<tr>
<td>Return to operator labor and mgt.</td>
<td>4,500</td>
<td>5,605</td>
<td>6,017</td>
</tr>
</tbody>
</table>

¹ Same organization and returns obtained when the profit maximizing model was used.
² Acres include summer fallow for wheat and sorghum where required.
⁴ Includes depreciation, interest, taxes, insurance, and other miscellaneous expenses

Solutions to the Profit Maximizing Model

On the 1,600-acre farm the organization of enterprises obtained using the profit maximizing model was found to be identical with organization 3 as computed by the minimum cost model. Also, the return obtained by the profit maximizing model, $17,662, was equal to the highest return obtained with the cost minimizing model. In the general case, these two organizations must be identical since the return maximizing organization is also the least cost method of obtaining that specific and maximum level of return. In fact this is the only (assuming the solution is unique) organization of enterprises capable of achieving the maximum return and both models must give identical results at this point.

The profit maximizing model is of no value in determining organiza-
tions which yield less than the maximum returns possible. One could determine farm organizations preferred by some farmers, such as a cash-grain farm, with the profit maximizing model by omitting all livestock enterprises. But this analytical procedure is still based on a management strategy that maximizes profits rather than on one that minimizes costs. To pursue this further, an additional constraint was added to the profit maximizing model limiting all livestock enterprises to the zero level. The solution gave the maximum return possible from a completely cash-grain organization, and as would be expected, this return was higher than the return from organization 1 but variable costs were substantially higher. Organization 1 then is the minimum cost method of obtaining a $16,145 return and is a cash-grain setup. This does not mean that it is the maximum return obtainable without livestock.

**Use of Minimum Cost Programming Methods**

Minimum cost linear programming techniques appear to be useful to the research worker in several ways. First, we would expect many farmers to tend away from the organization developed by the profit maximizing model, especially in an area of high production variability like the Great Plains. For example, some elements of uncertainty are related to the profitability and psychology of capital use. Farmers tend to view any increase in capital use as involving a “greater risk” since with a given set of expectations, such increased usage increases the range of possible outcomes. This factor may be considered distinct from equity considerations which may tend to limit capital usage in the form of borrowed funds. In addition, farmers may charge a high labor reservation price, discount returns from certain enterprises, and face certain firm-household relationships that make farm profit maximization a secondary goal. Variation in these factors in the real world results in a large range of organizations on farms that appear alike from the standpoint of available resources and alternative production possibilities.

The minimum cost technique is a method of considering such a set of

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*This statement applies only to the profit maximizing model used in this article. This model allows gross income to vary and selects an organization of enterprises resulting in higher net returns over variable costs than any other organization, given the specified fixed resources. An alternative model, which could be called a “restricted” profit maximizing model would require the programmer to specify the maximum level of gross income to be obtained. If items are defined in such a way that gross income = net returns + variable costs for each activity, this “restricted” profit maximizing model would give the same solutions as the minimum cost model discussed in this article.

behavioristic constraints for a particular group of farmers. To the extent that these constraints can be expressed mathematically they can be incorporated in the profit maximizing model and the solution obtained directly. However, the minimum cost technique will tell us something about the effect of such behavioristic constraints without expressing them explicitly since as these constraints become more important to the individual farm manager, his organization would tend more toward the minimum cost solutions with lower net returns. Hence the minimum cost approach is more general in that it indicates possible reactions of many farmers without complete knowledge of their behavioristic attitudes. The research worker is, of course, still left with the decision as to which level of return a particular operator would attempt to obtain. Even so, this decision can be made within a much more formal framework with the minimum cost analysis.

The minimum cost linear programming models can also be used for obtaining solutions to other problems to which the profit maximizing models are not well adapted. For example, minimum cost linear programming models can be used to determine the minimum complement of resources needed to earn specific levels of income. This is accomplished essentially by including a land buying activity in the minimum cost model discussed in this article, and allowing the "fixed costs" to vary as called for by the size of operation.

A second example is using a cost minimizing analysis to study economies of size. This requires the analysis of each size of firm at several levels of output (return) to provide information necessary to construct a short-run average cost curve for each firm. (Such a curve for the 1,600-acre wheat-sorghum farm in northwest Kansas is presented in Figure 2.) A long-run average cost curve constructed as an envelope of such short-run average cost curves for different sizes of farms would give information about economies of size for such firms. The Farm Production Economics Division, ERS, USDA, is currently conducting studies of this kind for major types of farming regions in the United States.

Summary

The use of a minimum cost linear programming model has been illustrated and discussed on a real farm situation. Some of the relationships between this model and a profit maximizing model in solving management and organizational problems on a specific farm have been explored, and solutions from the two different models compared.

The minimum cost strategy merits consideration by researchers concerned with predicting farmer response to given situations. In general,
farmers may not have the selection of an organization of enterprises that maximizes income as a primary goal. This is due to such factors as uncertainty, imperfect knowledge of the production process itself, and because of certain behavioral constraints. These factors may be merely a definitional problem—that is no problem exists when the researcher considers such constraints in defining the profit maximizing model. However, when unidentified behavioral constraints do exist, they cause discrepancies between the researchers' "profit maximizing organization" and the actual farm organizations. In such cases it appears that minimum cost linear programming techniques could give additional understanding of the relationships involved, whether used alone or in combination with profit maximizing techniques.

For purely predictive research, the minimum cost analysis furnishes some knowledge of a wider range of possible outcomes than the profit maximizing used alone. As a farm management tool, comparison of the minimum cost and the maximum profit solutions would provide a farmer with an idea of how much his preferences are costing him. In both areas,
minimum cost linear programming techniques should be fertile fields for further research.

Minimum cost linear programming also is applicable to areas in production economics to which profit maximizing models are not well suited. One example of this is studies of the minimum resources required for certain levels of income. Another is the use of minimum cost linear programming techniques to generate cost curves for use in studies of economics of farm size.